Verba volant, mathematica manent: Formalizing psychological theories as models

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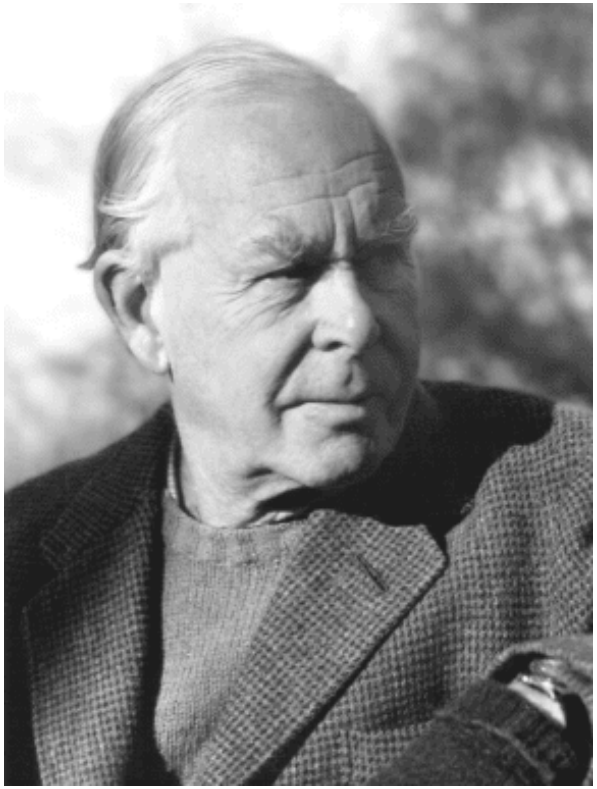
Attachment theory



John Bowlby (1907 - 1990)

Mary Ainsworth (1913 – 1999)

Attachment theory

A typical day in the life of a PhD student:

- Review the literature
- Research questions
- Study design
- Collect the data
- Analyse the data
- Interpret the results

Data analysis

Linear regression

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip} + \epsilon_i$$

Data analysis

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Good approach in an **exploratory study**

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but . . .

Data analysis

Good approach in an **exploratory study**

but . . .

we are not testing our research hypotheses.

Model comparison

Sources of inspiration

- **McElreath, R. (2016). Statistical Rethinking**: A Bayesian Course with Examples in R and Stan

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- **McElreath, R. (2016). Statistical Rethinking**: A Bayesian Course with Examples in R and Stan
- **Aust, F. (2017). A Conceptual Introduction to Mathematical Modeling of Cognition** (link)
- **Dablander, F. (2019). Bayesian modeling using Stan: A case study** (link)

Modeling in cognition[1]

Relation between practice and reaction times:

- **Exponential model** assumes a constant learning rate

$$f_e(N) = \alpha + \beta e^{-rN}$$

- **Power model** assumes diminishing returns

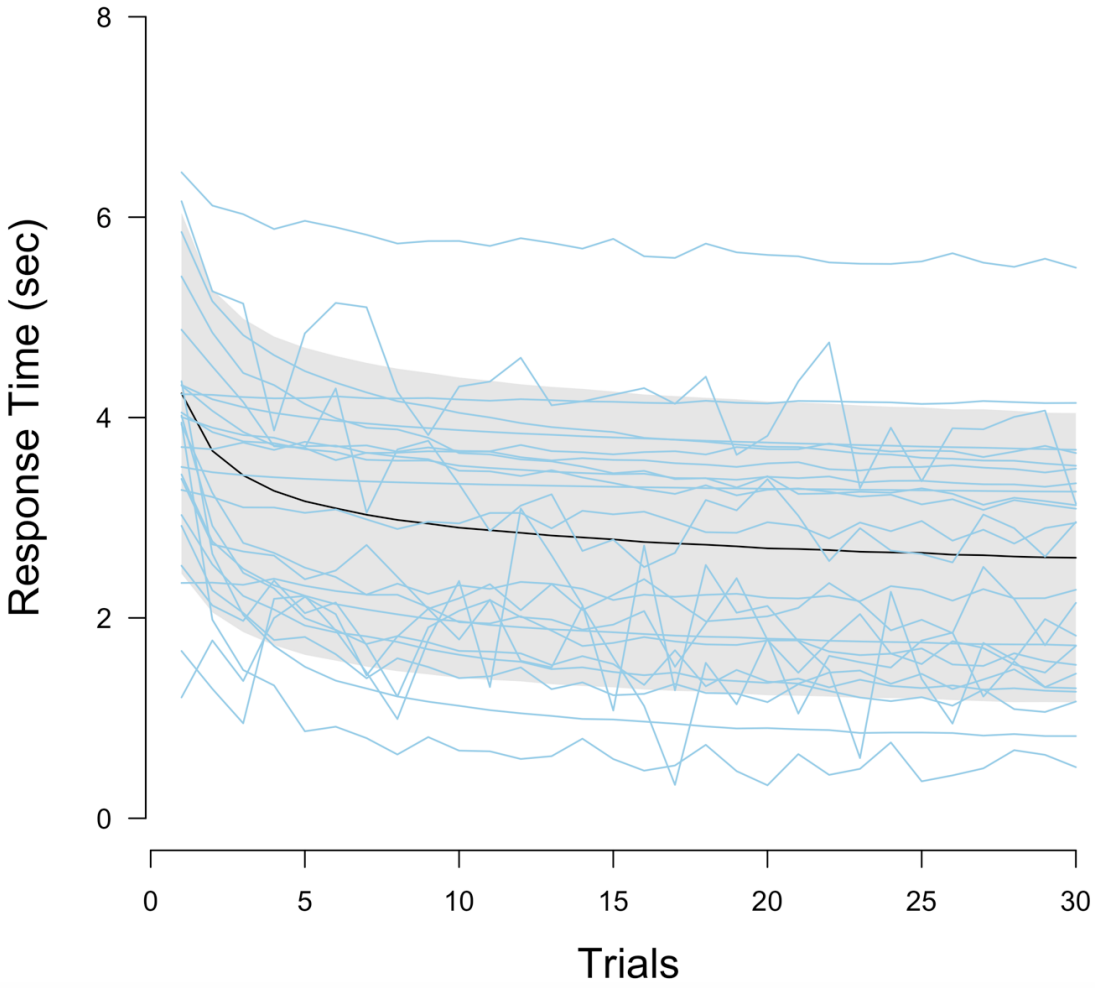
$$f_p(N) = \alpha + \beta N^{-r}$$

β is the *learning gain* and r is the *learning rate*

[1] Example from Dablander, F. (2019)

Modeling in cognition[1]

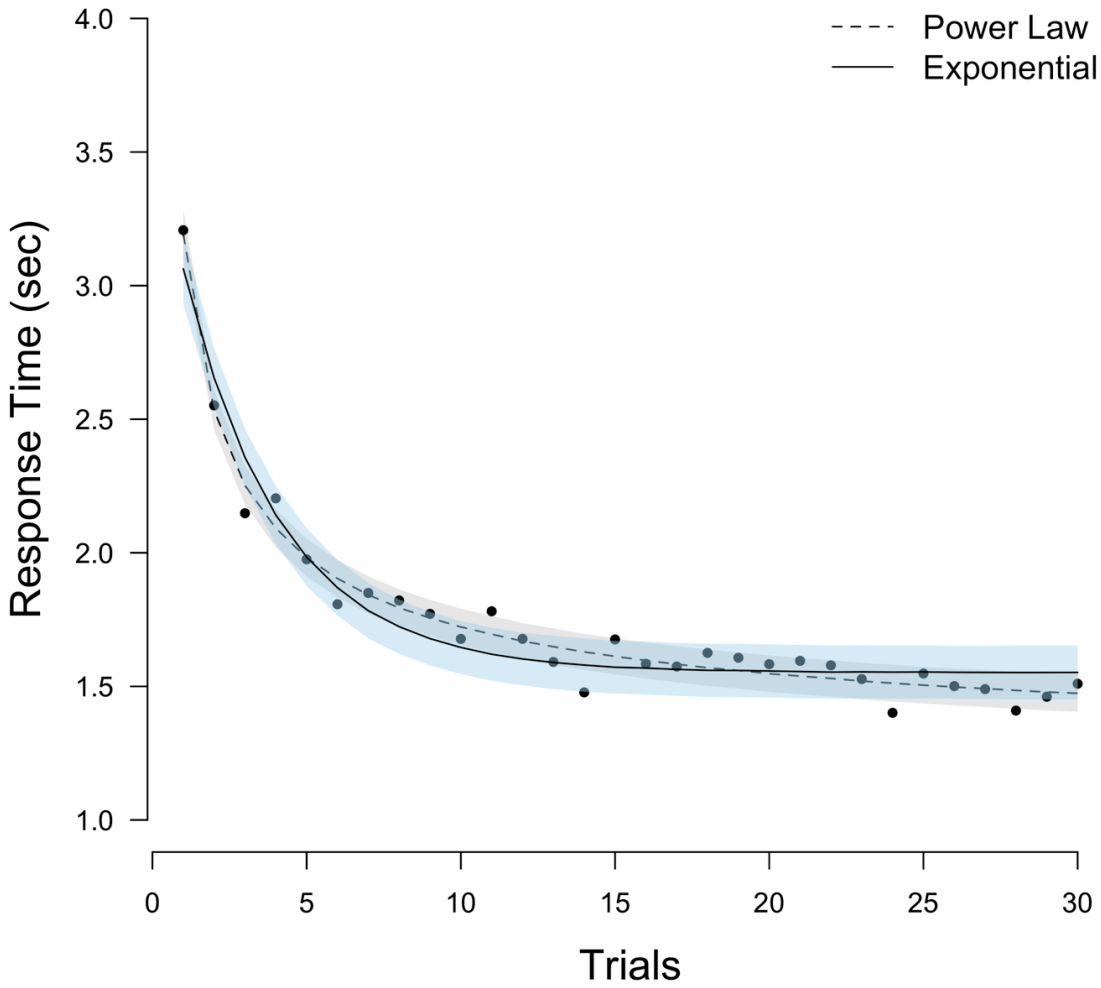
Prior predictions



[1] Example from Dablander, F. (2019)

Modeling in cognition[1]

Posterior predictions



[1] Example from Dablander, F. (2019)

Advantages of modeling

Formalize theories using mathematical models allows us to:

- **Specify** and **clarify** underlying assumptions of verbal theories
- Focus on the **data generating process** rather than on the description of the observed data
- Obtain **predictions** that can be used to evaluate the models and can inform future studies

Father in attachment theory

Theoretical perspectives

Main theoretical perspectives regarding the role of mother-child and father-child attachment:

1. **Monotropy theory**

Theoretical perspectives

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Main theoretical perspectives regarding the role of mother-child and father-child attachment:

1. **Monotropy theory**
2. **Hierarchical theory**
3. **Independent theory**
4. **Interaction theory**

Formalizing the models

Dependent variable:

- Externalizing problems (`Ext`)
- Internalizing problems (`Int`)

Formalizing the models

Dependent variable:

- Externalizing problems (`Ext`)
- Internalizing problems (`Int`)

Independent variable:

- Mother attachment (`Mother`)
- Father attachment (`Father`)

Attachment is considered as a dichotomous variable (secure = 0; insecure = 1)

Monotropy model

Model formula

- Externalizing problems

$$\texttt{Ext} = \alpha + \beta_{Ext;M}\texttt{Mother\_1}$$

- Internalizing problems

$$\texttt{Int} = \alpha + \beta_{Int;M}\texttt{Mother\_1}$$

Monotropy model

Model prior

Psychol Bull. 2016 Apr;142(4):367-99. doi: 10.1037/bul0000029. Epub 2015 Nov 30.

Representational and questionnaire measures of attachment: A meta-analysis of relations to child internalizing and externalizing problems.

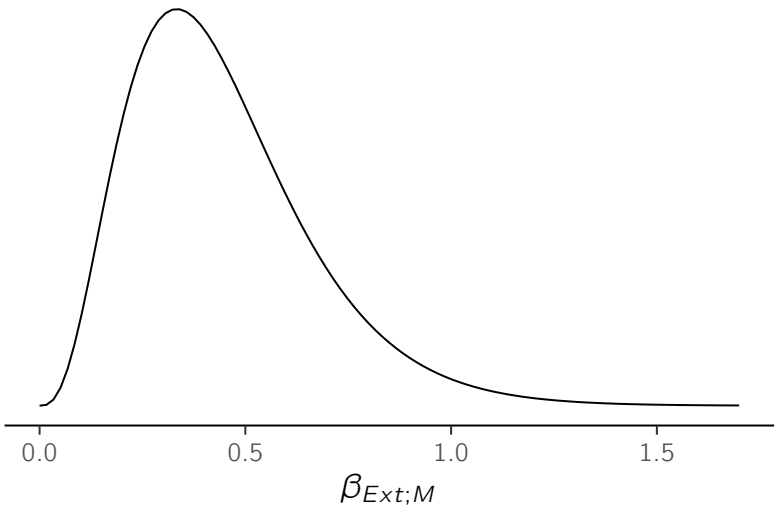
Madigan S[1], Brumariu LE[2], Villani V[3], Atkinson L[3], Lyons-Ruth K[4].

- When secure attachment was compared with insecure attachment, modest associations with **internalizing behavior** (165 studies; 48,224 families; **d** = **.58**; **95%CI**[**.52**–**.64**]) were found
- Attachment and **externalizing behavior** were also associated (116 studies; 24,689 families; **d** = **.49**; **95%CI**[**42**–**.56**])

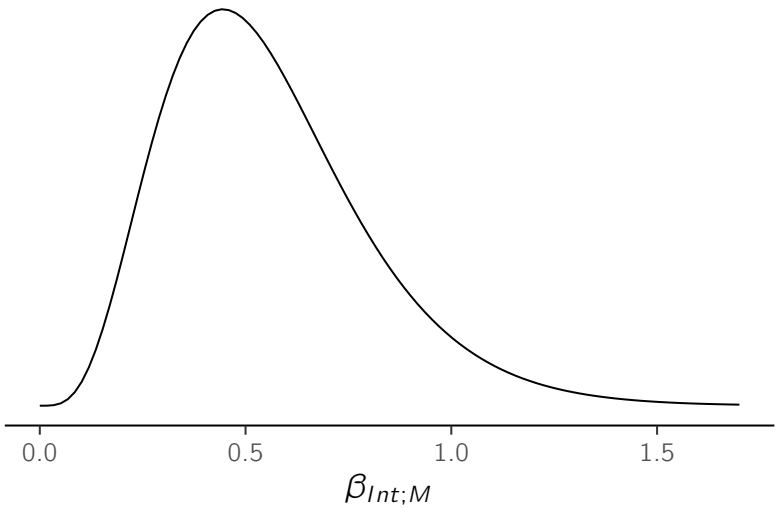
Monotropy model

Model prior

$Gamma(scale = 4, rate = 9)$



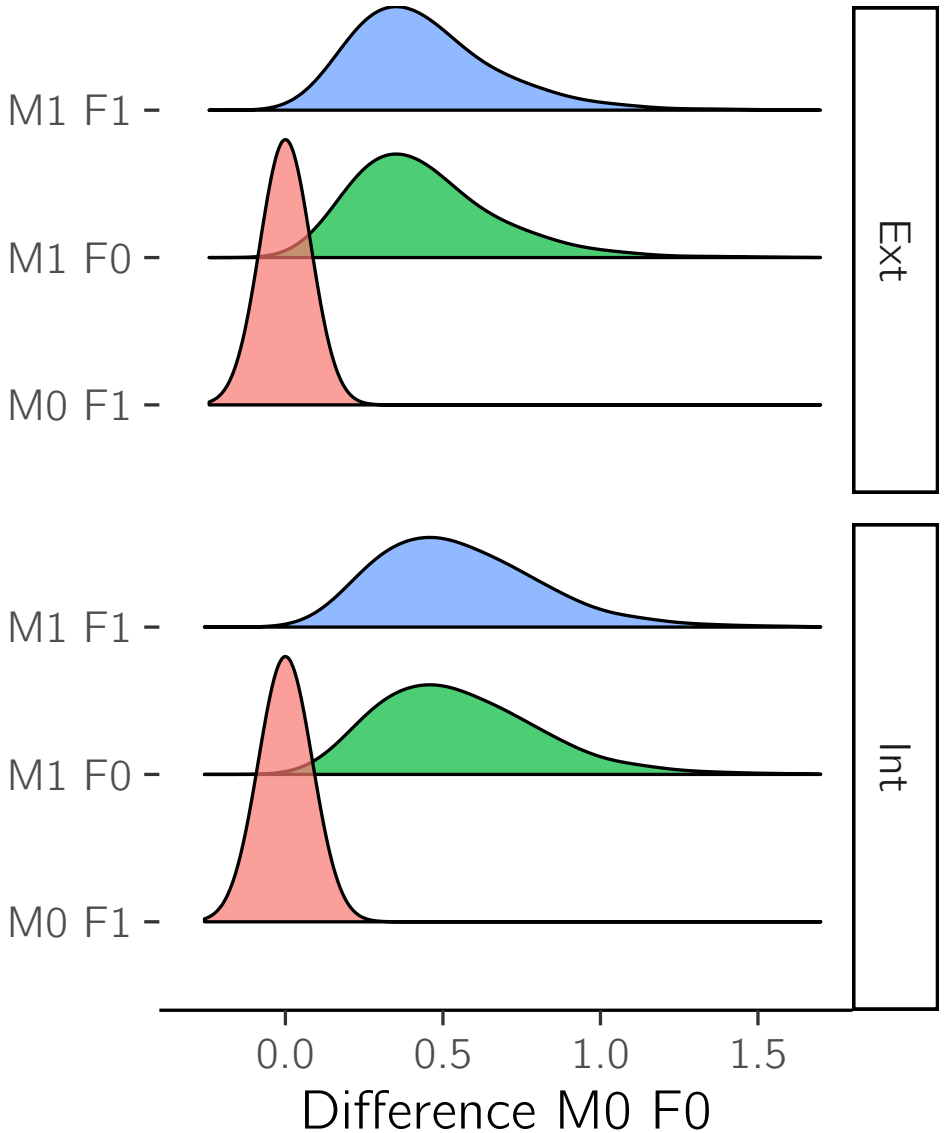
$Gamma(scale = 5, rate = 9)$



		89%HDI	
Parameter	**Mean**	**Lower**	**Upper**
$\beta_{Ext;M}$	0.44	0.11	0.76
$\beta_{Int;M}$	0.56	0.18	0.92

Monotropy model

Prior predictions



Hierarchical model

Model formula

- Externalizing problems

$$\texttt{Ext} = \alpha + \beta_{Ext;M}\texttt{Mother\_1} + \beta_{Ext;F}\texttt{Father\_1}$$

- Internalizing problems

$$\texttt{Int} = \alpha + \beta_{Int;M}\texttt{Mother\_1} + \beta_{Int;F}\texttt{Father\_1}$$

As Father is supposed to contribute less than the mother, we define

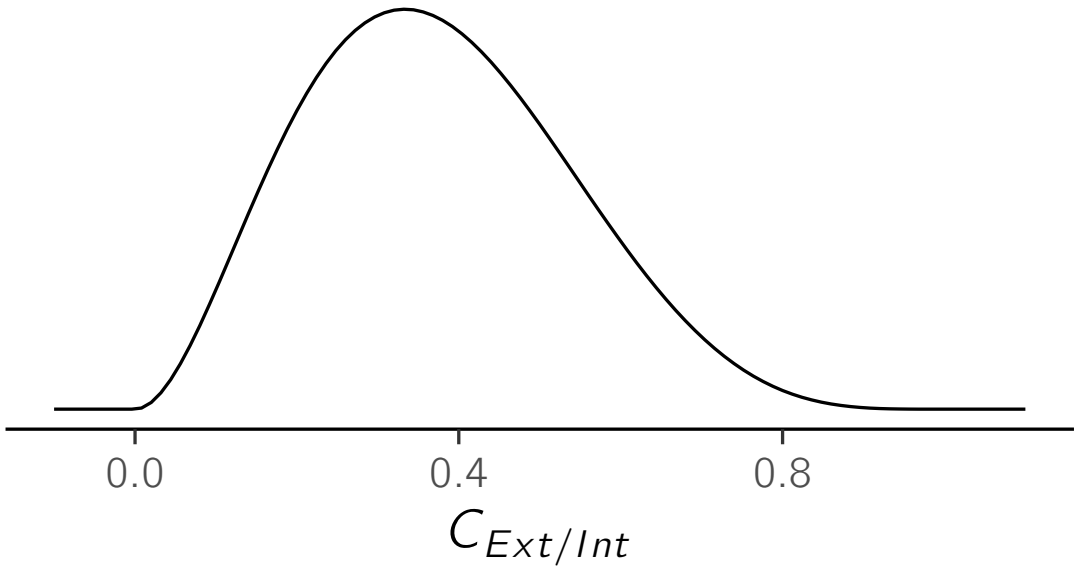
$$\beta_{Ext;F} = C_{Ext} \times \beta_{Ext;M}, \qquad \beta_{Int;F} = C_{Int} \times \beta_{Int;M}.$$

Where C_{Ext} and C_{Int} are bounded between 0 and 1

Hierarchical model

Model prior

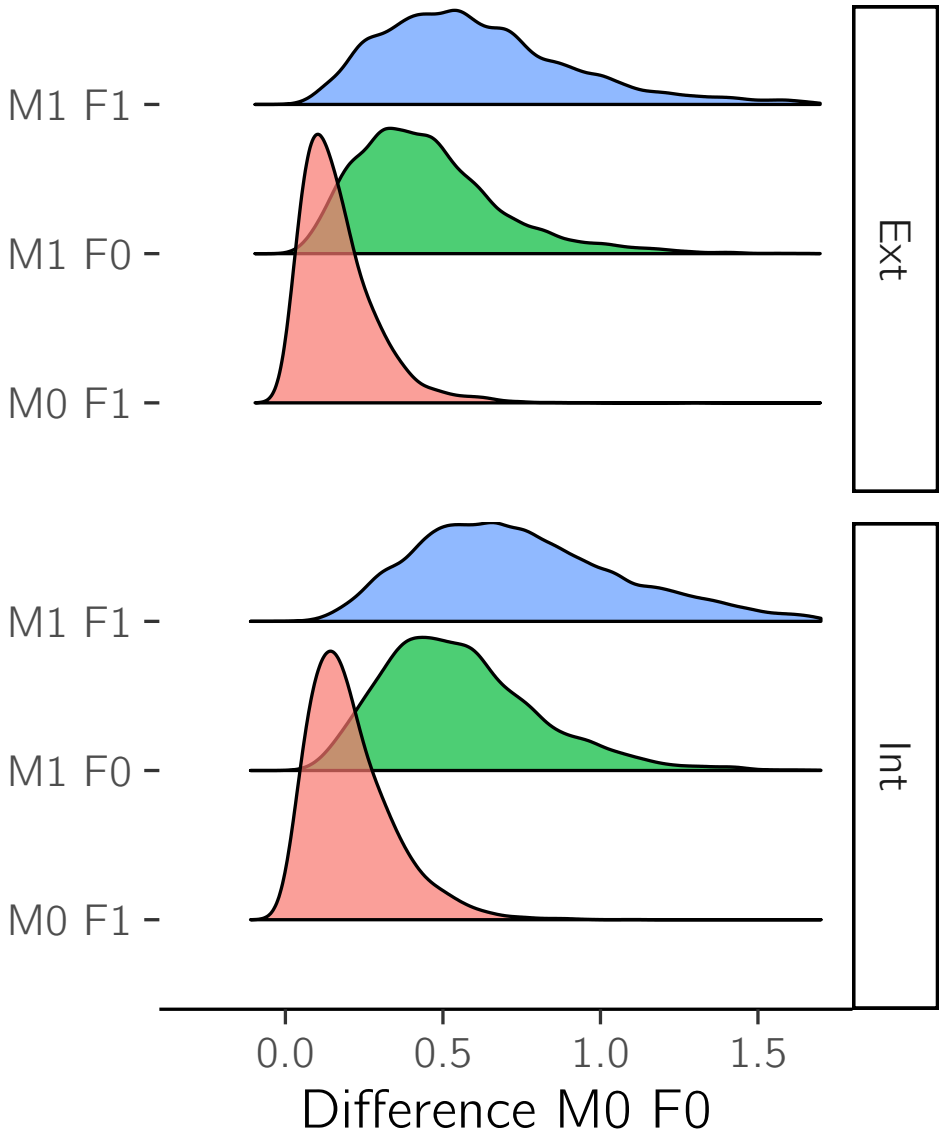
$$Beta(\alpha = 3, \beta = 5)$$



		89%HDI	
Parameter	**Mean**	**Lower**	**Upper**
$C_{Ext/Int}$	0.38	0.11	0.62

Hierarchical model

Prior predictions



Independent model

Model formula

- Externalizing problems

$$\texttt{Ext} = \alpha + \beta_{Ext;F}\texttt{Father\_1}$$

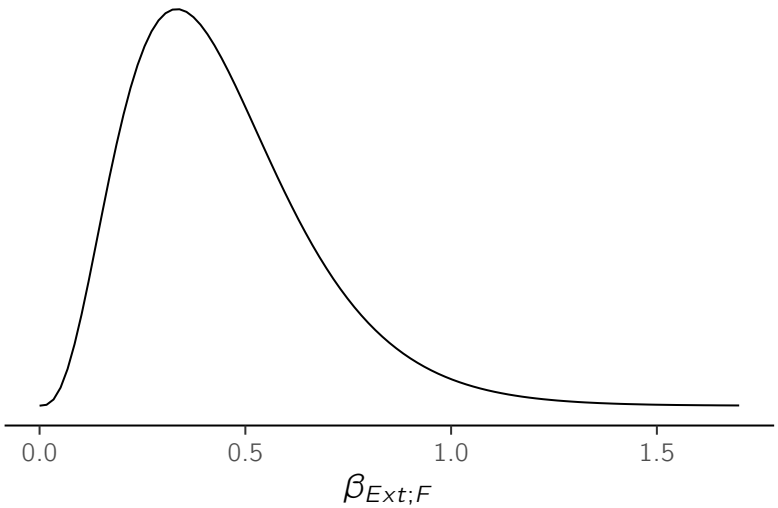
- Internalizing problems

$$\texttt{Int} = \alpha + \beta_{Int;M}\texttt{Mother\_1}$$

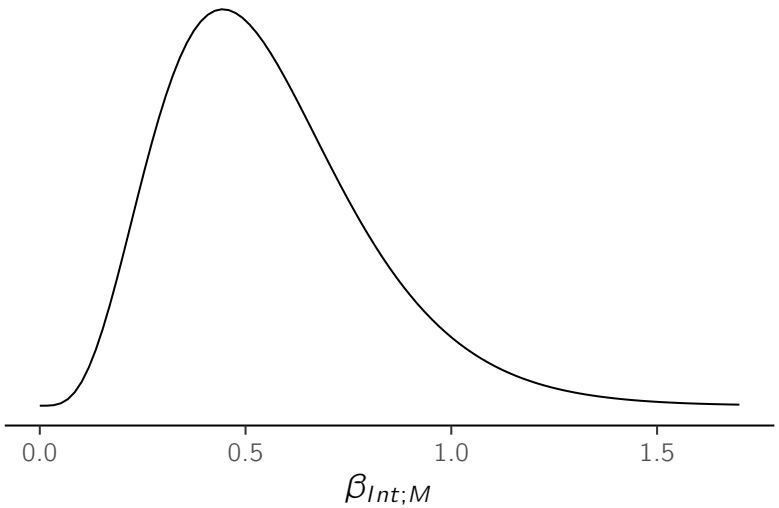
Independent model

Model prior

$Gamma(scale = 4, rate = 9)$



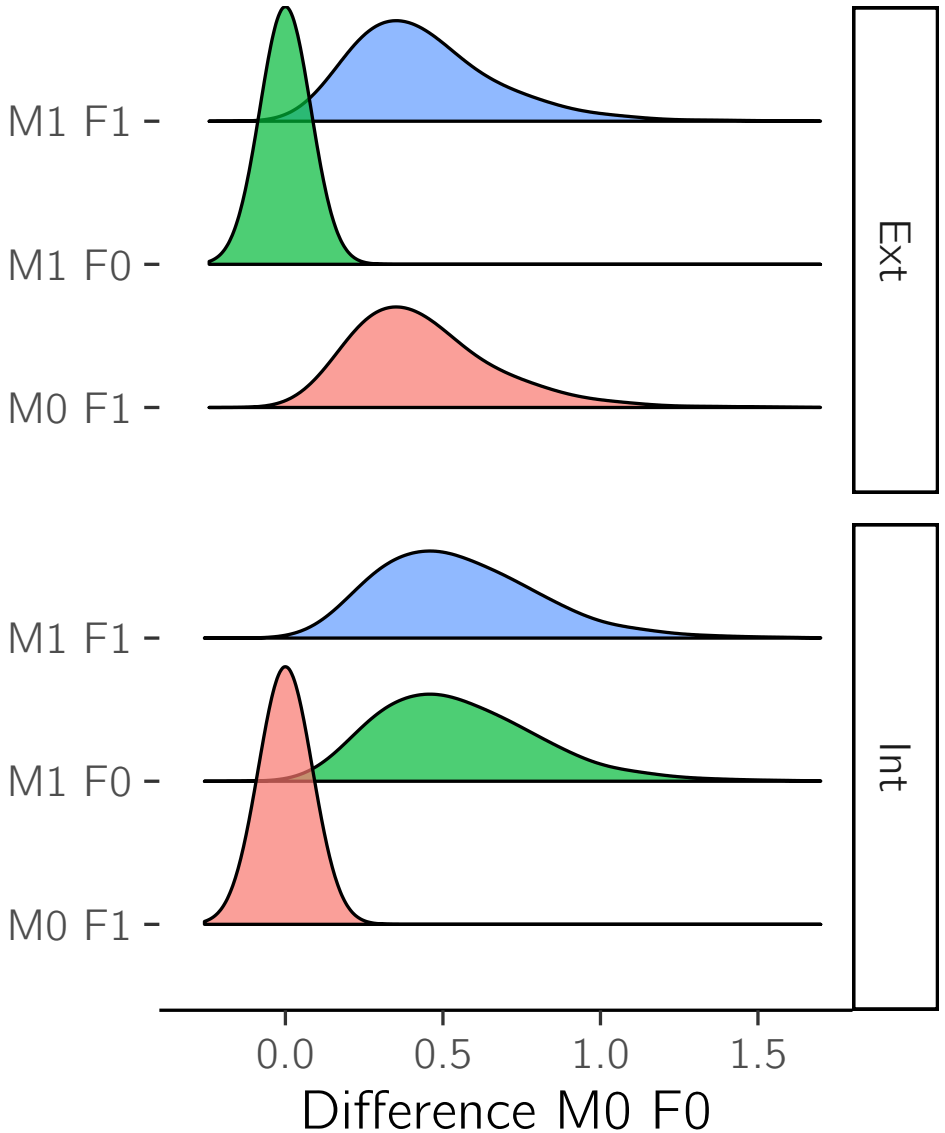
$Gamma(scale = 5, rate = 9)$



		89%HDI	
Parameter	**Mean**	**Lower**	**Upper**
$\beta_{Ext;F}$	0.44	0.11	0.76
$\beta_{Int;M}$	0.56	0.17	0.91

Independent model

Prior predictions



Interaction model

Model formula

- Externalizing problems

$$\texttt{Ext} = \alpha + \beta_{Ext;M}\texttt{Mother\_1} + \beta_{Ext;F}\texttt{Father\_1} + \beta_{Ext;Int}\texttt{Interaction}$$

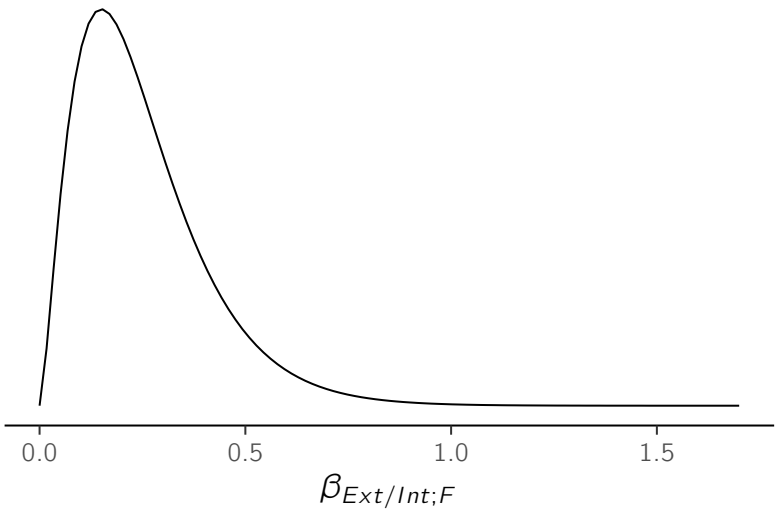
- Internalizing problems

$$\texttt{Int} = \alpha + \beta_{Int;M}\texttt{Mother\_1} + \beta_{Int;F}\texttt{Father\_1} + \beta_{Int;Int}\texttt{Interaction}$$

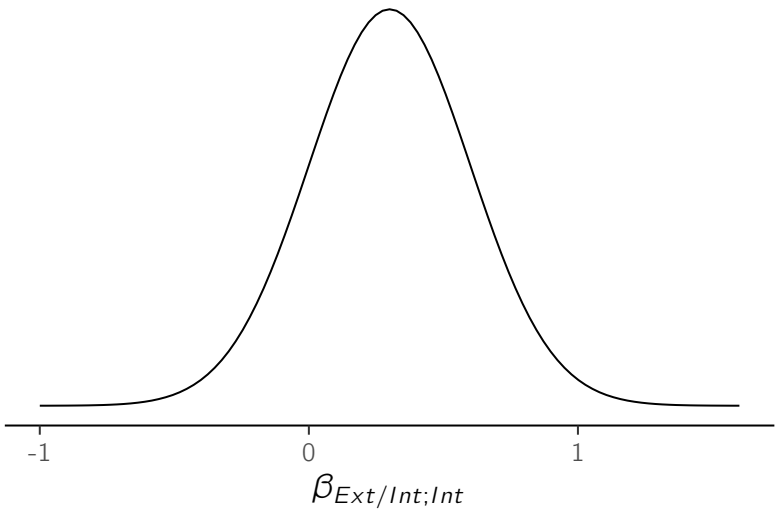
Interaction model

Model prior

$Gamma(scale = 2.5, rate = 10)$



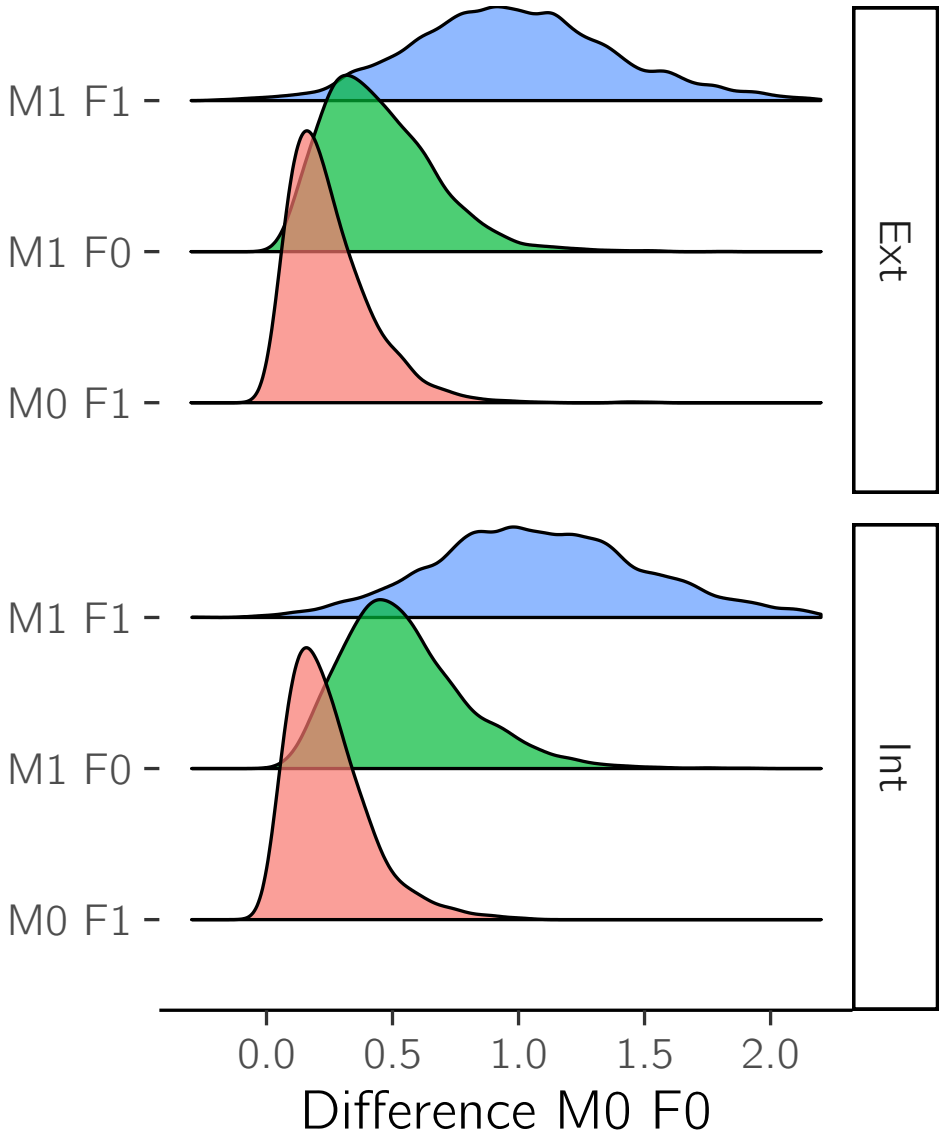
$Normal(mu = .3, sd = .3)$



		89%HDI	
Parameter	**Mean**	**Lower**	**Upper**
$\beta_{Ext/Int;F}$	0.25	0.02	0.46
$\beta_{Ext/Int;Int}$	0.30	-0.18	0.78

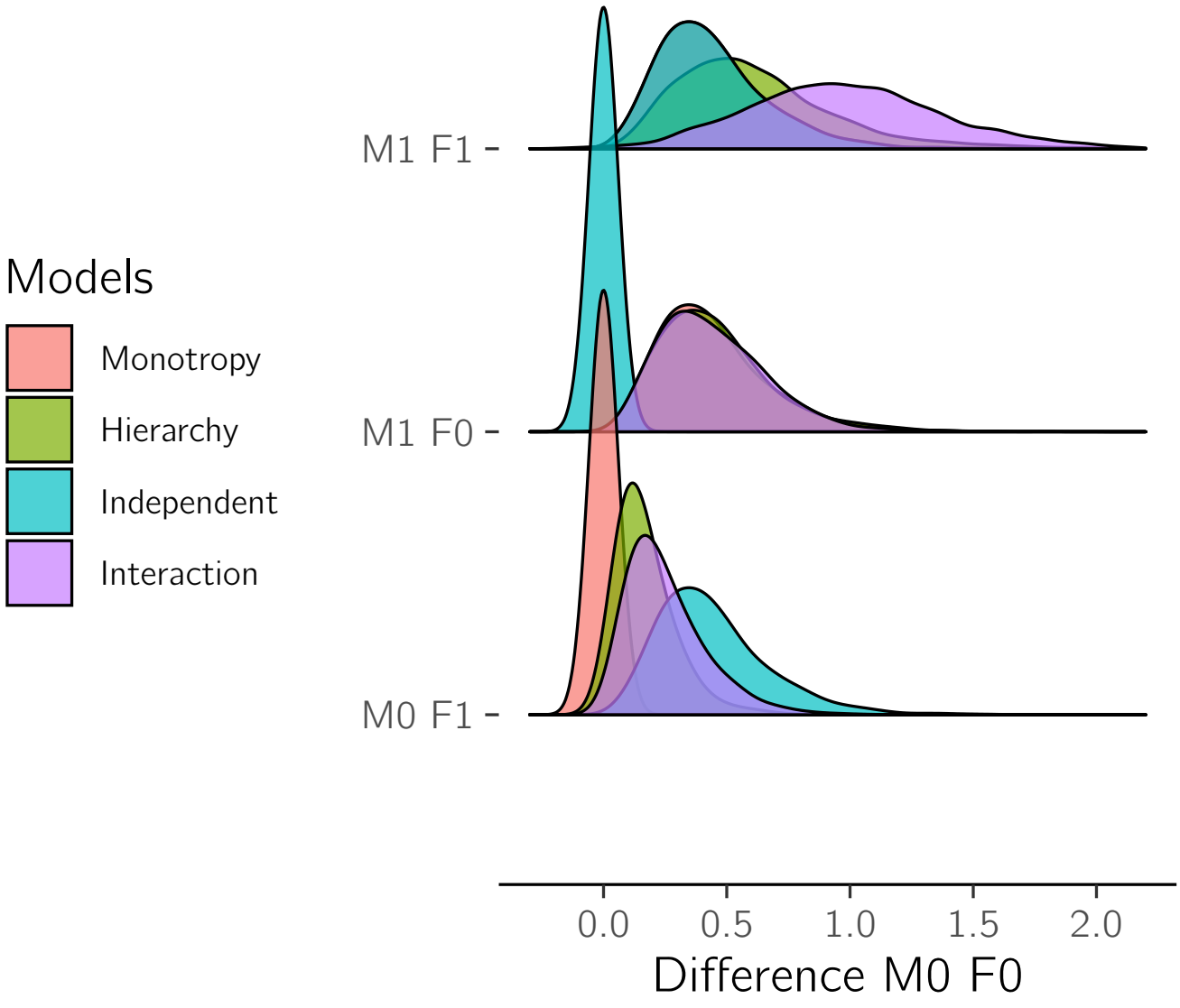
Interaction model

Prior predictions



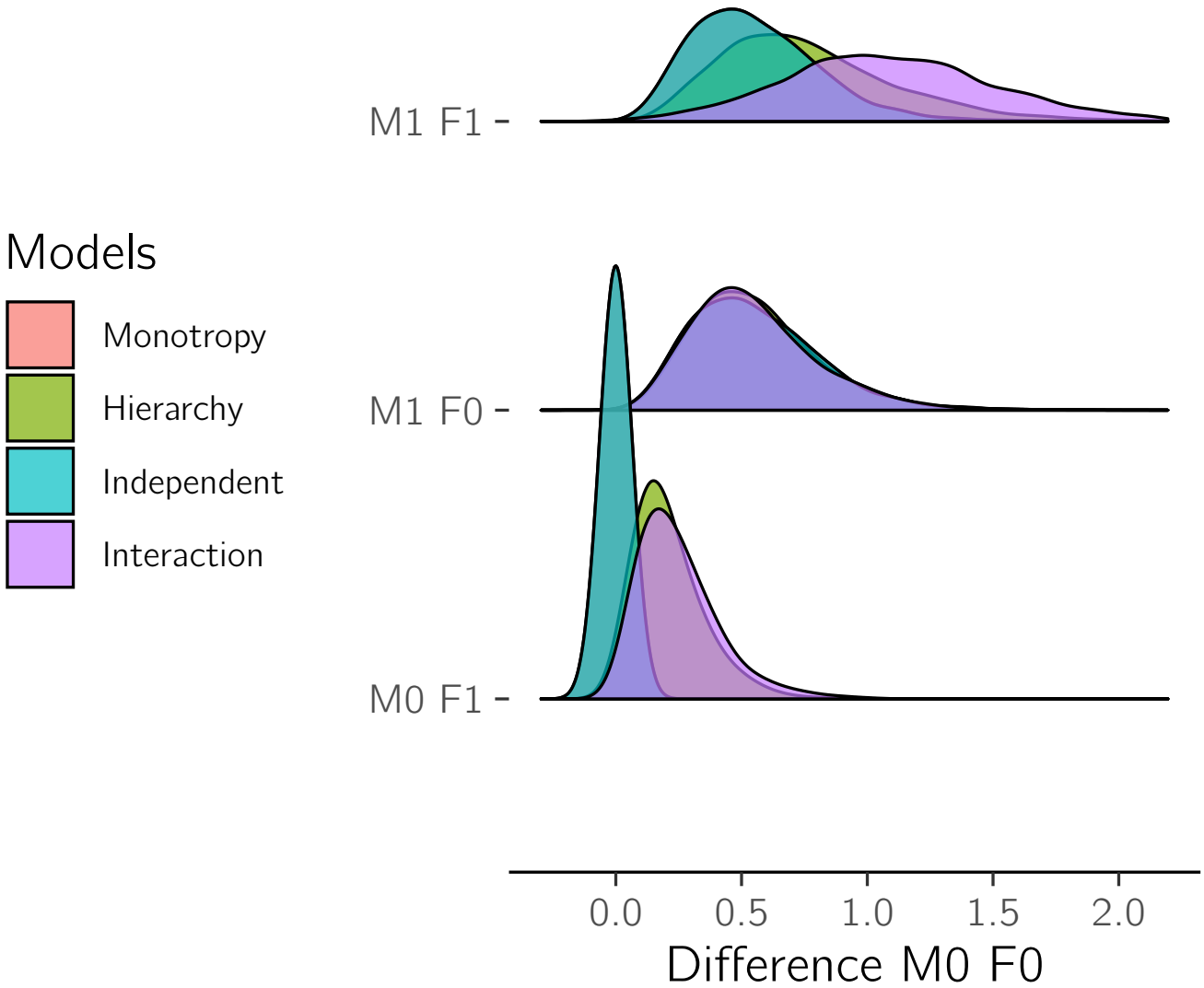
Overview models

Prior predictions Externalizing



Overview models

Prior predictions Internalizing



Conclusions

Final considerations

Formalizing models...

- ...is a long process
- ...requires **multiple expertises**
- ...requires **multiple sources of information**
- ...is a **creative** process
- ...is an **iterative** process

Final considerations

Many **subjective decisions** are involved in the process. However, as long as they are transparently reported and discussed, they become **reasonable choices**

This allows us to move the debate to another level (i.e., choices of the model or of the priors) and it helps to **highlight current limits** and it opens to **future improvements**

Thanks!

"All models are wrong, but some are useful"
(Box, 1978)